Q: An archer standing on a 15° downwards slope shoots an arrow 20° above the horizontal. How far down the slope does the arrow hit if it is shot with a speed of 50 m/s from 1.75 m above the ground?

A: One way to answer this question is to write x vs. y equations for the ground and for trajectory of the arrow's and find the intersection. Setting up a coordinate system with the origin at the archer's feet, we the equation of the ground is given by

$$y = -x\,(\tan 15^\circ)$$
$$y = -0.2679x$$

Let us now find equations $x(t)$ and $y(t)$ equations for the position of the arrow as a function of time. Here are the things we know:

$$x_0 = 0 \text{ m}$$
$$y_0 = 1.75 \text{ m}$$
$$v_{x0} = 50\cos 20^\circ \text{ m/s} \approx 46.98 \text{ m/s}$$
$$v_{y0} = 50\sin 20^\circ \text{ m/s} \approx 17.10 \text{ m/s}$$

The horizontal position $x(t)$ of the arrow is given by

$$x = 46.98t$$

Solving for t, we have

$$\rightarrow t = \frac{x}{46.98}$$

Meanwhile, the horizontal position of the arrow $y(t)$ is given by

$$y = 1.75 + 17.10t - 4.9t^2$$

By eliminating t from this equation, we get the arrow's trajectory

$$y = 1.75 + 17.10\left(\frac{x}{46.98}\right) - 4.9\left(\frac{x}{46.98}\right)^2$$

$$y = 1.75 + 0.364x - 0.00222x^2$$

Finally, we set the vertical positions of the ground and arrow equal to each other, giving

$$-0.2679x = 1.75 + 0.364x - 0.00222x^2$$

We can solve this with the quadratic formula.

$$\boxed{x = 287m}$$